Supporting Clear and Concise Mathematics Language

Instead of That, Say This

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Juan, a child with a mathematics disability, is learning about addition and subtraction of fractions. Juan’s special education teacher, Mrs. Miller, has tried to simplify language about fractions to make fractions easier for Juan. During instruction, she refers to the “top number” and “bottom number.” At the end of chapter test, Juan reads the problem: “What’s the least common denominator of 1/2 and 1/3?” Juan answers, “1.”

Upon returning his test, Mrs. Miller asks Juan how he arrived at his answer, and learns that because he didn’t know what denominator meant, he used the word least to choose the number that was “least.” Mrs. Miller explains that denominator is the formal term for the “bottom number.” Juan exclaims, “I know how to find the least common bottom number!”

Mrs. Miller did not intend to make mathematics confusing for Juan; she tried to make mathematics easier. But, in simplifying her language without connecting this informal language to formal mathematics language, she did Juan a disservice.

Children with disabilities, like Juan, perform lower in mathematics than their peers without disabilities, and this gap widens from ages 7 to 13 (Wei, Lenz, & Blackorby, 2013). Of even greater concern is that fifth-grade children with mathematics disabilities continue to perform in the bottom quartile of their grade in high school (Shalev, Manor, & Gross-Tsur, 2005). This trend leads educators to ask the question: With multiple tiers of instruction, why do low-performing children in the elementary grades continue to struggle with mathematics?

One influence contributing to this trend may be the imprecise use of mathematics language. Educators may not interpret mathematics as a second (or third) language for children, when, in fact, all children are mathematical-language learners (Barrow, 2014). The numerals, symbols, and terms that explain mathematics concepts and procedures are plentiful and complex.

The language of mathematics, especially vocabulary terms, is necessary for understanding mathematics in oral and written forms (Ernst-Slavit & Mason, 2011; Riccomini, Smith, Hughes, & Fries, 2015). Mathematics vocabulary is often difficult for children because many terms have meanings in general English and meanings specific to mathematics (Rubenstein & Thompson, 2002; Schleppegrell, 2007). For example, factor could mean a contributing element (e.g., one factor contributing to mathematics difficulties is vocabulary that requires language code switching) or two or more numbers multiplied together to produce a product (10 and 12 are factors of 120). Even within the mathematical definition of factor, produce and product may have multiple meanings.

Language plays an important role in learning mathematics. In Juan’s case, his special education teacher was trying to make fractions easier for Juan to understand, but because of the simplified language Mrs. Miller used in instruction, Juan did not understand grade-level questions presented with mathematical vocabulary. Just as there are rules in mathematics that expire in later grade levels (e.g., multiplication always results in a bigger number; Karp, Bush, & Dougherty, 2014), there are mathematics terms (e.g., bottom number) that help children only temporarily. One way for educators to improve Juan’s mathematics performance is by developing an understanding of and sensitivity to mathematics language that facilitates conceptual and procedural understanding. Children should learn mathematics skills in accurate contexts that provide a solid foundation on which to build more complex skills in later grades. Therefore, teaching language that is mathematically correct and holds true across grade levels can help children generalize mathematics language that supports accurate and conceptual understanding of mathematics. We discuss five domain areas within the elementary Common Core State Standards for Mathematics (CCSS; National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010), and not only explain why it is important to use clear and concise mathematics language but also which terms to use in working with students.

**Counting and Cardinality**

A solid foundation for success in mathematics begins with understanding that numbers have given values. Correct counting requires stable order (i.e., a sequence of counting words), one-to-one
correspondence, and cardinality. Cardinality refers to the understanding that the last number word of a counting sequence (e.g., "one, two, three, four, five" for a set of five toy cars) represents the quantity in the set. Given that these skills provide the foundation for future success in calculation (e.g., addition; Noël, 2009), it is not surprising that counting and cardinality constitute almost one third of the kindergarten CCSS. It is important, however, that children are exposed to counting and cardinality language in a way that allows for integration of higher-level, complex mathematical understanding. In terms of language, educators must be mindful of how they talk about numbers within the counting sequence (see Figure 1).

Our language suggestions pertain to the counting sequence. Purposeful language may promote students to extend mathematics understanding as they learn new concepts that complement the continuum of learning. For example, although teaching the concept of negative integers in kindergarten is not developmentally appropriate, teachers can present counting and cardinality using a number line that extends in both directions, requiring children to locate 1 (or another number) when counting. Teachers should not indicate that any number is “first” when counting or looking at a number line because this indicates there are no numbers less than the “first” number. Instead, refer to counting as starting at a specified number. In a similar manner, educators should not indicate an end to the counting sequence (e.g., 10). Many children have difficulty with the teen numbers and beyond (Geary, 2000), and as too many mathematics trade books about counting finish with 10 (Powell & Nurnberger-Haag, 2015), it is important to use language that supports numeral and quantity understanding beyond 10. Two problematic examples pertaining to 10 are songs with lyrics “7, 8, 9, and the last one’s 10” or “8, 9, and 10.” Using accurate language that promotes counting as a continuum on a number line provides children with an accurate understanding of numbers so that they can flexibly incorporate new, complex understanding when it is developmentally appropriate.

Number and Operations in Base 10

As children develop counting and cardinality skills, they also acquire an understanding of number and operations in base 10. This begins with composition and decomposition of two-digit numbers. For example, the number 15 can be represented as one bundle of 10 and five 1s or as fifteen 1s. Children also learn to compare two-digit numbers using symbols and words (i.e., >, =, and <). Not only do children need to develop a flexible understanding of numbers, but they need a solid foundation in mathematics language to discuss numbers. For example, children need to know the following terms in order to compare numbers: greater than, less than, place value, digit, more, and fewer. Children’s understanding of place value builds in complexity with each grade, and so do the language demands. By fifth grade, children must understand and describe the relationship between each place value (e.g., a digit in the hundreds place represents 10 times as much as the digit in the tens place), compare decimals to the thousandths, and multiply or divide whole numbers and decimals using place-value strategies. Unfortunately, many children struggle to develop proficiency in this domain (DeWolf, Grounds, Bassok, & Holyoak, 2014), and deficiency in number and base-10 operations greatly affects understanding of number and operations in base 10.

Figure 1. Counting and Cardinality

Instead of . . . Say . . .

1 is the first number

Problem: 1 is not the first number. The number line extends infinitely in both directions. Referring to 1 as “the first number” causes confusion over understanding zero, negative integers, and rational numbers.

Let’s start counting with 1 or 0
Solution: This accurately represents a conceptual understanding of counting and number sense. Numbers do not start at a particular place, but rather you choose to begin counting at 0, 1, or another integer.

And the last one is 10

Problem: This suggests that 10 is the final or highest number. As many children struggle with teen numbers, it is necessary to give opportunities to count beyond 10.

...8, 9, 10. We’ll stop counting there, but we could count more
Solution: Providing an indication that 10 is a temporary stopping point helps children understand there are numbers beyond 10.

...7, 8, 9, 10...
Solution: In mathematics, only use “and” when referring to the decimal point.

...7, 8, 9, and 10
Problem: The use of “and” suggests that 10 is the final or highest number.
children’s performance in other domains of mathematics.

When teaching this domain, incorporate language that supports place-value understanding and develops flexibility in mathematical thinking. Although it may seem easier to teach children chants, tricks, or rules for solving place value and computation problems, these approaches often use inaccurate language that does not support conceptual understanding. It is extremely important that educators model and maintain accurate language that facilitates conceptual understanding of place value.

It is also necessary to tie instruction to place-value understanding and developing flexibility in mathematical calculation. For comparison, educators should use accurate language, such as greater than or less than. These terms replace the common description that “the alligator eats the bigger number.” In this example, no conceptual understanding is established with eats. Also, numbers are not bigger but greater or more. The term bigger can cause difficulty years later with addition and subtraction of positive and negative integers. The same goes for the term smaller. With calculations based on the four operations (i.e., addition, subtraction, multiplication, division), educators must be mindful of the language used to explain the operator symbols (+, −, ×, ÷), inequality symbols (<, >), and equivalence symbols (=, ≠). The plus sign means to “add,” but it does not signal “plussing.” The plus sign may be explained as putting together, but this definition is short-lived as children are encouraged to start with a set and add on to the set (Fuchs et al., 2009). Another symbol that causes difficulty in later grades is the equal sign. Educators could use language, such as the same as or a balance, to help children understand the equal sign as a balance between two sides of an equation (e.g., Powell, Driver, & Julian, 2015). Educators should avoid saying the equal sign means “write your answer” or “compute.” As children use symbols to perform multidigit computation, educators must be aware of using language that supports the concept of regrouping (e.g., regroup) rather than the notational procedure (e.g., borrow).

Educators may not interpret mathematics as a second (or third) language for children, when, in fact, all children are mathematical language learners.

Numbers and Operations With Rational Numbers

Numbers and operations with rational numbers address mathematics involving fractions, decimals, and percentages. Rational numbers tend to be one of the most problematic areas for children, and difficulty with rational numbers affects later mathematics learning (Hoffer, Venkataraman, Hedberg, & Shadle, 2007). Any difficulty with rational numbers is concerning given research linking rational number achievement to later success in mathematics (e.g., Bailey, Hoard, Nugent, & Geary, 2012). It is not alarming, however, when we consider how properties of rational numbers may differ from properties of whole numbers. Language terms taught early about whole numbers may no longer apply when working with fractions or decimals (see Figure 3).

Many of the language suggestions for rational numbers pertain to how teachers communicate fractions to children using words or images. For example, a fraction is a unique number whose magnitude or value can be identified on a number line. A fraction comprises numerals (e.g., 3 and 8) that create a single number (e.g., 3/8). The denominator of a fraction represents the equal parts of the relative whole (i.e., whole, group, set, measurement), not just parts. Children commonly divide a rectangle into eight unequal parts and shade three of the parts to represent 3/8 without an understanding that the whole must be divided into equal parts. If an educator merely added equal to parts every time the denominator is mentioned in instruction, children might develop a better understanding of wholes and parts. Educators must also provide exposure to fraction concepts that do not fit the meaning of equal parts in a whole. For example, a fraction can be parts of a set or a position on a number line.

Educators should use the technical terms numerator and denominator under most circumstances and relegate top- and bottom-number language to descriptions of the position of the numerator and denominator. Top number and bottom number are strictly terms that describe the position of the numbers in the numerator and denominator, and fractions are not
### What number is in the tens place?

Problem: This does not help the child understand place value. A number refers to the entire amount. For example, 243 is a number. The 4 in the tens place value is not a number, but rather a digit.

### Bigger number and smaller number

Problem: This is not mathematical language and it does not transfer to positive and negative integers.

### Equals

Problem: This term is often used to indicate that children write an answer.

### When adding, your answer is always bigger. When subtracting, your answer is always smaller.

Problem: This is not always true. When working with 0, rational numbers, or negative numbers, adding will not always produce a greater number and subtracting will not always produce a number that is less.

### Makes up or break apart

Problem: These informal terms are procedural and not the terms used in textbooks or on high-stakes assessments.

### Five hundred and twenty-nine

Problem: The word “and” should only be used to represent the decimal point (e.g., 325 is “3 and twenty-five hundredths”) or fractions (e.g., 3 ¼ is “3 and one-fourth”).

### The alligator eats the bigger number

Problem: Children do not learn how to read math expressions from left to right or understand the meaning of the greater-than (>) and less-than (<) symbols.

### What digit is in the tens place? What is the value of the digit 4 in the tens place?

Solution: This reinforces the conceptual understanding of place value and emphasizes that 4 is part of the number 243 with a value of 40.

### Five hundred twenty-nine

Solution: This is mathematically correct.

### Compose and decompose

Solution: Use the formal terms to describe composing or decomposing a number (e.g., “24 is composed of 2 tens and 4 ones”).

### Less than or greater than

Solution: Children learn how to read and write the inequality symbols and read equations correctly from left to right. Children also learn that < and > are two distinct symbols and not one symbol that switches back and forth.

### Number that is greater and number that is less

Solution: These terms are mathematically accurate and reflect the language in mathematics standards.

### the same as

Solution: This reinforces the equal sign as a symbol that indicates the quantities on both sides need to be the same.

### Ask children to predict and reason

Solution: Do not say rules that expire in subsequent grade levels because it leads to an erroneous understanding of addition and subtraction.

### Regroup or trade or exchange

Solution: This reinforces the conceptual understanding of regrouping ones into tens, tens into hundreds, and so on, or ungrouping hundreds into tens, tens into ones, and so on.
<table>
<thead>
<tr>
<th>Instead of . . .</th>
<th>Say . . .</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Numbers in the fraction</strong></td>
<td><strong>This fraction is a number</strong></td>
</tr>
<tr>
<td>Problem: Language suggests that each part of a fraction (i.e., numerator, denominator) is a separate and independent number, instead of digits (or series of digits) that comprise a fraction.</td>
<td>Solution: A fraction is a number in itself and has a magnitude on a number line. A fraction is not two separate numbers.</td>
</tr>
<tr>
<td><strong>Top number</strong> and <strong>bottom number</strong></td>
<td><strong>Numerator</strong> and <strong>denominator</strong></td>
</tr>
<tr>
<td>Problem: This suggests that the numerator and denominator are separate and independent numbers.</td>
<td>Solution: A fraction is a number with a specific magnitude that can be represented on a number line. Although a fraction may have different parts, these parts do not work in isolation but rather contribute to one number: the fraction.</td>
</tr>
<tr>
<td><strong>2 over 3</strong></td>
<td><strong>Two-thirds</strong></td>
</tr>
<tr>
<td>Problem: This communicates the location of the digits but not the actual number or magnitude.</td>
<td>Solution: This is accurate and communicates the magnitude of the number.</td>
</tr>
<tr>
<td><strong>Line</strong></td>
<td><strong>Fraction bar</strong> or <strong>slash</strong></td>
</tr>
<tr>
<td>Problem: Calling the fraction bar a “line” is inexact vocabulary.</td>
<td>Solution: The fraction bar or slash plays an important role in communicating the divisional relationship between the numerator and denominator.</td>
</tr>
<tr>
<td><strong>Reduce</strong></td>
<td><strong>Rename</strong> or <strong>find an equivalent fraction</strong></td>
</tr>
<tr>
<td>Problem: This term (as in “reduce to the lowest term”) suggests the result is less in quantity.</td>
<td>Solution: The quantity represented by the magnitude of fraction does not change. The only change is with the digits used to communicate that magnitude.</td>
</tr>
<tr>
<td><strong>Three point four</strong></td>
<td><strong>Three and four tenths</strong></td>
</tr>
<tr>
<td>Problem: Reading a decimal as “point” does not support the conceptual understanding of place value of the magnitude of the decimal.</td>
<td>Solution: This reinforces place value and supports understanding of magnitudes, values, and when to use each symbol.</td>
</tr>
<tr>
<td><strong>Move the decimal point over</strong></td>
<td><strong>Demonstrate process within Base 10</strong></td>
</tr>
<tr>
<td>Problem: This language communicates what is superficially occurring, the action. The language does not promote conceptual understanding when multiplying or dividing by tens.</td>
<td>Solution: Helps with understanding the process of multiplying by tens, hundreds, and so on.</td>
</tr>
<tr>
<td><strong>Three out of four</strong></td>
<td><strong>Three to four</strong></td>
</tr>
<tr>
<td>Problem: When talking about ratios, this language is incorrect because it does not communicate the ratio of one number to another, but rather one number to the whole.</td>
<td>Solution: Although a minor change in language, the meaning is very different and communicates the ratio of one number to another.</td>
</tr>
</tbody>
</table>
always presented with a fraction bar (e.g., \( \frac{1}{4} \)), which negates these terms. In a similar way, describing a fraction as a number over a number does not help children understand that a fraction is a single quantity. Students’ misunderstanding that fractions consist of two separate numbers separated by a line may lead to errors such as adding across the top number and adding across the bottom number when adding fractions. As children learn to reduce a fraction to lowest terms, some children believe this means the value of the fraction changes; a better choice is to describe determining an equivalent fraction in simplest form, which eliminates the need to use the term reduce.

Decimals communicate similar information as fractions but are based on powers of 10. As such, the language used to read a decimal can support the relationship. For example, reading 5.4 as “five and four tenths” naturally connects fractions to decimals by sharing how fractions can be written as decimals. Casual language, such as saying point (e.g., “five point four”) as the placeholder for the decimal point, spills over to discussion on how decimals are manipulated (e.g., “move the decimal point over”) instead of building conceptual understanding of the base-10 system. Another language consideration is introduced with out of and ratios. Many educators use out of to describe the parts of a whole (e.g., “three out of four” for \( \frac{3}{4} \)), but with ratios, out of does not convey the same meaning (e.g., 3.2 is not “three out of two” but “three to two”).

Geometry

Children typically start school with a basic understanding of shapes (Clements & Sarama, 2000). Within the CCSS, geometry appears as a domain area at kindergarten and all subsequent grades through eighth grade. Much of geometry in the elementary grades focuses on two-dimensional (2-D) and three-dimensional (3-D) shapes. In the late elementary grades, children are expected to understand lines and angles and how these relate to properties of shapes and coordinate planes. Children with mathematics difficulty struggle with geometry concepts through high school (Dobbins, Gagnon, & Ulrich, 2014); therefore, it is necessary to provide a consistent and strong geometry background to children across grade levels. Often, general vocabulary is used to describe geometric concepts, yet children are expected to interpret formal geometric vocabulary. Educators should show the connection between informal and formal terms (see Figure 4).

Many of the issues around language with geometry pertain to preciseness of vocabulary. At the earliest grades, educators may use informal names for shapes, like ball for circle, when a ball is actually a sphere. Children must understand the term circle and use it across grade levels, so introducing this term early and applying it consistently is necessary. The same is true for square and rectangle; a square is a rectangle, but a rectangle is not always a square. Mathematical language accuracy is also important for understanding that the space between intersecting lines is an angle and not a corner. Children do not measure corners, but they do measure angles.

In the late elementary grades, language used to describe 2-D shapes can change for 3-D shapes, so educators must explicitly help children identify these changes and connect the concepts. A cube may be described as having six sides, but these sides are faces. The sides are actually edges, and edges meet at vertices, not points. When calculating the volume, two of the faces of the cube are bases. Another language concern is around the term same. An educator may use same to describe figures that are similar, congruent, and symmetrical. Using same may be helpful in the short term, but as children are asked to find similar and congruent shapes, same does not help with this task.

As children learn transformations in the early elementary grades, educators often describe these as flips, slides, and turns. Although these terms describe the action of a transformation, children in the later elementary and middle school grades must be familiar with the formal terms of reflection, translation, and rotation. Specificity with the term is necessary for children to have gained adequate exposure to the term for practice within textbooks and on high-stakes assessments. In a similar manner, shapes do not shrink or stretch. Instead, these are dilations of a shape.

Measurement and Data

The domain area of measurement and data is mentioned specifically in the CCSS across kindergarten through fifth grade. At kindergarten, children are expected to describe length and weight and compare objects with measureable attributes. Measurement of objects continues across the elementary grades with a focus on standard and nonstandard units of measurement. In first grade, children start learning about telling time, and in second grade are introduced to money. Starting in third grade, measurement becomes intertwined with geometry as children measure perimeter and area of shapes, and continue with measurement of angles in fourth grade and measurement of volume in fifth grade. Representing data is a theme across the elementary grade levels. Figure 5 highlights important components of language when educators teach measurement and data concepts.

Similar to the geometry domain area, many of the language issues with measurement relate to being precise with language and not using certain terms interchangeably. When educators introduce telling time, the clock hands

One way to support children and promote progressive understanding of mathematics is to use precise and accurate language.
### Figure 4. Geometry

#### Instead of . . .

**Box or ball**

Problem: With early descriptions of shapes, children use terms that relate to real-life objects. This is permissible, but formal language should also be reinforced.

**Square** (for any rectangular shape)

Problem: A square has four equal, straight sides, and four right angles. A square is a rectangle, but a rectangle is not necessarily a square.

**Corner**

Problem: This general vocabulary term is not mathematically accurate.

**Side or angle** (to describe 3D shapes)

Problem: A 2-D shape uses straight sides, and the sides meet to form angles. This is not true for 3-D shapes.

**Point** (for 3-D figures)

Problem: This term (e.g., “reduce to the lowest term”) suggests the result is less in quantity.

**These are the same shape**

Problem: This description is too vague.

**These shapes are the same**

Problem: This description is too vague.

**These halves are the same**

Problem: This statement does not convey conceptual meaning.

**Flips, slides, and turns**

Problem: Although these terms may help children remember the action of a transformation, this vocabulary is not used on assessments.

**Stretch or shrink**

Problem: These terms may help children remember the action of a transformation, but this vocabulary is not used on assessments.

#### Say . . .

**Square/rectangle or circle**

Solution: Use the formal language of shapes to confirm informal language.

**Rectangle**

Solution: This helps children distinguish between square and rectangle terminology.

**Angle**

Solution: Reinforce that an angle is the space between two intersecting lines.

**Edge, face, or vertex/vertices**

Solution: This reinforces conceptual understanding that 2-D and 3-D figures are different.

**Vertex**

Solution: This is the endpoint where two or more line segments or rays meet.

**These shapes are similar**

Solution: This description is more accurate and precise; shapes are similar when the only difference is in size.

**These shapes are congruent**

Solution: This precisely describes similar shapes that are the same size.

**These are symmetrical**

Solution: This statement describes the reflection of a shape.

**Reflections, translations, and rotations**

Solution: These are the correct mathematical terms.

**Dilation**

Solution: This is the proper mathematical term.
should be referred to as the minute hand and hour hand so that children can understand which hand indicates minutes and which indicates hours. As children compare quantities, it is important to use language correctly. For example, “Gabe’s amount is less than Marta’s” is grammatically correct over “Gabe’s amount is fewer than Marta’s.” Numbers should not be described as bigger but instead as greater because greater is associated with quantity. For detailed measurement, length refers to the measurement of a side or edge. Educators may use weight and mass interchangeably, yet weight refers to how much an object weighs down on a surface, whereas mass refers to
the matter within an object. On a high-stakes assessment, a child may be presented with a pictorial representation of a liquid measuring cup filled with a liquid. The question may ask about the capacity of the cup and the volume of the liquid. In order to understand the task, the child must understand that capacity and volume have different meanings but similar calculations. For interpretation of data, educators should be specific with the types of data representations (e.g., chart, graph, picture, pictograph) so children can create appropriate representations of data.

Implications for Practice
Taking steps to prevent mathematics difficulty for children is important. One way to support children and promote progressive understanding of mathematics is to use precise and accurate language embedded within teaching strategies that progress and generalize across standards and grade levels. In this article, we shared several examples of ways to adjust common errors in mathematics language; however, the lists provided do not encompass all possible language errors and faux pas. Many of our language suggestions help to support conceptual understanding of mathematical concepts, and this is often troublesome for children with mathematics disabilities. The clear and concise mathematics vocabulary we describe can be incorporated into existing evidence-based practices and instruction with ease. For example, when using manipulatives to demonstrate the concepts of multiplication and regrouping, educators can focus on describing base-10 blocks as hundreds, tens, and ones and reinforcing regrouping or exchanging. Because clear and concise mathematical language sets children up for success, educators in subsequent grade levels may not have to reteach so many misconceptions related to language and rules (Karp et al., 2014).

In addition, it is important for special educators to consult the standards and curricula across grade levels to understand language and expectations for future successes. For example, a third-grade educator may benefit from looking at not only third-grade standards but also fourth-, fifth-, and sixth-grade standards. Teaching children so that they are successful in mathematics requires that educators plan for not only short-term success but long-term success. We encourage educators to use the information shared in this article to attend to the importance of mathematics language in instruction and evaluate personal use of correct mathematics language to support longitudinal learning.

References


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